Technical Comments

Comments on "Aerodynamic Penetration and Radius as Unifying Concepts in Flight Mechanics"

M. C. Smith*
Michigan State University, East Lansing, Mich.

Nomenclature

g = acceleration of gravity
D = $C_D\frac{1}{2}\rho V^2 A$ = aerodynamic drag
L = $C_L\frac{1}{2}\rho V^2 A$ = aerodynamic lift
s_D = $2m/\rho A C_{D_{av}}$ = aerodynamic penetration
s_L = $2m/\rho A C_{L_{av}}$ = aerodynamic radius
ρ = air density
y = distance of lateral drift
C_D = drag coefficient
V = flight speed
C_L = lift coefficient
n = L/W = load factor
R = range or turn radius
A = reference area for drag coefficient
φ = roll angle
t = time
m = vehicle or projectile mass
W = mg = vehicle weight
z = vertical distance
v = wind speed
ψ = yaw angle (Fig. 1)

Subscripts

av = average value
0 = initial value
vac = vacuum

Two additional applications of the concepts discussed in Ref. 1 can be described as wind drift and lateral maneuver. Wind drift is of interest in diverse studies such as small arms ballistics, parachute drops and unguided re-entry. Lateral maneuver is of particular interest in air-to-air missile and interceptor aircraft performance studies and in guided re-entry.

An example of wind drift can be taken from small arms ballistics. In this instance the flight is assumed to be horizontal with a horizontal wind normal to the flightpath. The projectile is either fin or spin stabilized which causes the longitudinal axis of the projectile to be aligned with the relative wind as shown in Fig. 1. Thus, the drag force has a component normal to the initial direction of motion of the projectile. For small drift the equation of drift motion is

$$D\psi = m\ddot{y} \tag{1}$$

or

$$C_D\tfrac{1}{2}\rho V^2 A(v - \dot{y}/V) = m\ddot{y} \tag{2}$$

The value of $V(t)$ can be obtained by assuming an average C_D and that the drift does not influence this variable. The differential equation is

$$m(dV/dt) = -C_D\tfrac{1}{2}\rho V^2 A \tag{3}$$

and the solution is

$$V = \frac{V_0}{1 + (V_0 t/s_D)} \tag{4}$$

Substitution of (4) into (2) and integration yields

$$y = v\left\{t - \frac{\ln[1 + (V_0 t/s_D)]}{V_0/s_D}\right\} \tag{5}$$

The time of flight, in vacuum, to a range R, is

$$t_{vac} = \frac{R}{V_0} = \frac{1}{V_0}\int_0^t V dt = \frac{\ln[1 + (V_0 t/s_D)]}{V_0/s_D} \tag{6}$$

Thus

$$y = v[t - t_{vac}] \tag{7}$$

which reveals that the drift of the projectile is proportional to the difference between the time of flight in the atmosphere and the time of flight that would occur in a vacuum. It is noted that this result is well known to small arms ballisticians on an empirical basis alone. Using the expression

$$dR = V dt \tag{8}$$

further results can be obtained as

$$V = V_0 e^{-(R/s_D)} \tag{9}$$

$$t = s_D/V_0(e^{R/s_D} - 1) \tag{10}$$

and

$$y/R = v/V_0[s_D/R(e^{R/s_D} - 1) - 1] \tag{11}$$

where V, t, and y are the velocity, time of flight, and drift distance occurring at the range R. It is noted that Eq. (11) gives in first approximation

$$y/R = 0 \tag{12}$$

and in second approximation

$$y = (v/V)(R^2/s_D) \tag{13}$$

showing the relations of the ratio of wind-drift velocity to initial velocity and of range and aerodynamic penetration in determining the wind drift. The "drop" of a projectile in a gravitational field is also of interest, but the second approximation to this, using Eqs. (8) and (9), is

$$z = \int_0^t\int g dt = \frac{gR^2}{V_D{}^2} \tag{14}$$

which is independent of s_D.

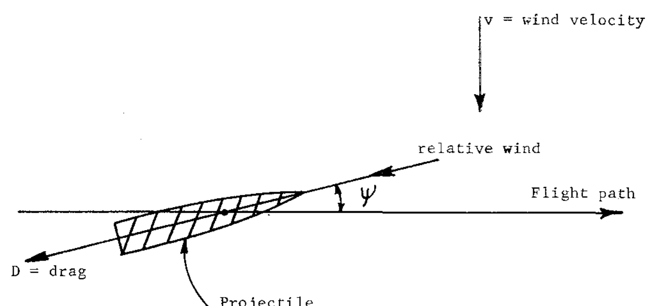

Fig. 1 Alignment of longitudinal axis of the projectile with the relative wind.

Received March 17, 1967.
* Assistant Professor, Mechanical Engineering.

An example of lateral maneuver can be taken from the aircraft intercept problem. The equations of motion for horizontal turn at constant flight speed are

$$L \cos\varphi = mg$$
$$L \sin\varphi = m(V^2/R) \quad (15)$$

Eliminating φ gives

$$R/s_L = (n^2/n^2 - 1)^{1/2} \quad (16)$$

where n is the load factor in the turn. It is seen from Eq. (16) that for $n \rightarrow \infty$ the turn radius equals the aerodynamic radius. Even for relatively low load factors, the turn radius only slightly exceeds the aerodynamic radius. The excess is only 6% for $n = 3$.

Reference

[1] Larrabee, E. E., "Aerodynamic penetration and radius as unifying concepts in flight mechanics," J. Aircraft **4**, 28–35 (1967).